

Lecture 6: Vector Valued Functions

Vector Valued Functions take a scalar as an input and output a vector.

$$\vec{F}(t) = f_1(t)\hat{i} + f_2(t)\hat{j} + f_3(t)\hat{k}$$

$$\vec{F}(t) = \langle f_1(t), f_2(t), f_3(t) \rangle$$

$f_1, f_2, + f_3$ are the "component functions" of \vec{F}

Ex. 1 Identify the component functions and domain of \vec{F}

$$\vec{F}(t) = t^3\hat{i} + \ln(3-t)\hat{j} + \sqrt{t}\hat{k}$$

$$f_1(t) = t^3 \quad f_2(t) = \ln(3-t) \quad f_3(t) = \sqrt{t}$$

domain of:

$$f_1(t) \text{ defined } \forall t \in \mathbb{R}$$

$$f_2(t) \text{ defined } \forall t \in (-\infty, 3)$$

$$f_3(t) \text{ defined } \forall t \in [0, \infty)$$

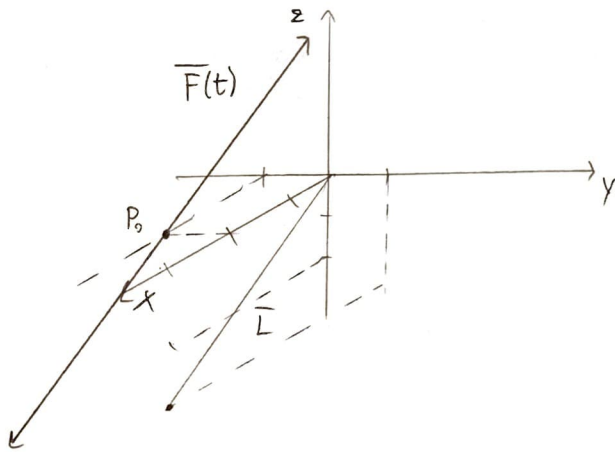
$$\therefore \vec{F}(t) \text{ defined } \forall t \in [0, 3)$$

Ex. 2 Sketch the curve of $\vec{F}(t) = \langle 2+3t, -1+t, -2t \rangle$

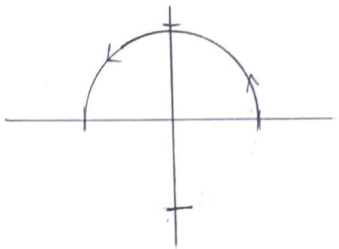
$$x(t) = 2+3t \quad y(t) = -1+t \quad z(t) = -2t$$

This is the equation of a line with $P_0 = (2, -1, 0)$ +

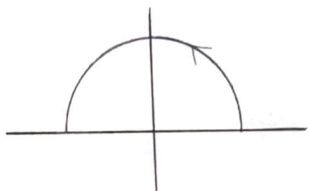
$$\vec{L} = \langle 3, 1, -2 \rangle$$



Ex. 3 $\vec{r}(t) = \langle \cos(t), \sin(t), 0 \rangle \quad t \in [0, \pi]$

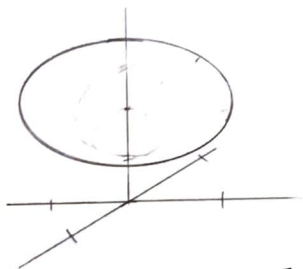


Ex. 4 $\vec{r}(t) = \langle \cos(2t), \sin(2t) \rangle \quad t \in [0, \frac{\pi}{2}]$

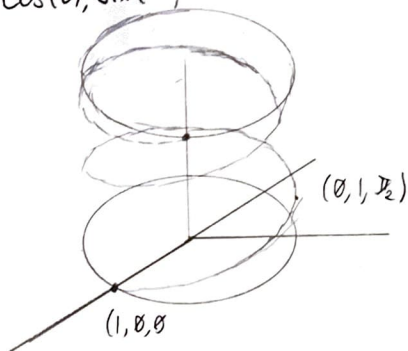


same curve different $\vec{r}(t)$

Ex. 5 $\vec{r}(t) = \langle \cos(t), \sin(t), 2 \rangle \quad t \in [0, 2\pi]$



Ex. 6 $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle \quad t \in [0, \infty)$



Ex. 7 $\vec{r}(t) = \langle (4 + \sin(20t)) \cos(t), (4 + \sin(20t)) \sin(t), \cos(20t) \rangle$

Yeah idk fan.
Sometimes we need computers. This is a toroidal spiral. Hella neat!