

Lecture 10

Functions of Several Variables

A function of several variables takes multiple inputs and outputs a single scalar.

Ex. 1

$$f(x, y) = x^2 + y^2, \quad g(x, y, z) = xyz, \quad h(x, y, z, w) = 2x + 4y^2 + z^5 + 9w^{1/3}$$

The domain is generally all values for which the function is well defined.

Ex. 2

Find the domain of $f(x, y) = \sqrt{9 - x^2 - y^2}$. Find the range

The square root function is defined for non-negative inputs.

$$9 - x^2 - y^2 \geq 0$$

$x^2 + y^2 \leq 9$ a circle of radius 3.

$$0 \leq \sqrt{9 - x^2 - y^2} \quad \text{and} \quad 9 - x^2 - y^2 \leq 9 \rightarrow f \leq \sqrt{9} = 3$$

$$0 \leq f \leq 3$$

The "graph" of a function is the collection of points $(x, y, f(x, y))$

Level Curves

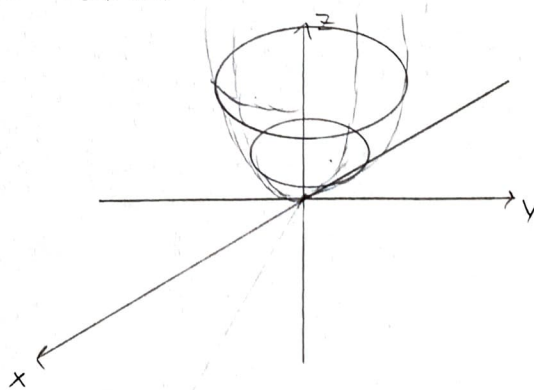
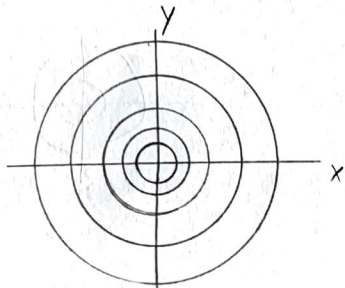
The set of points (x, y) that satisfy $f(x, y) = c$ is called the "level curve"

Think of elevation contour maps.

Ex. 3

Find the level curves of $f(x, y) = x^2 + y^2$

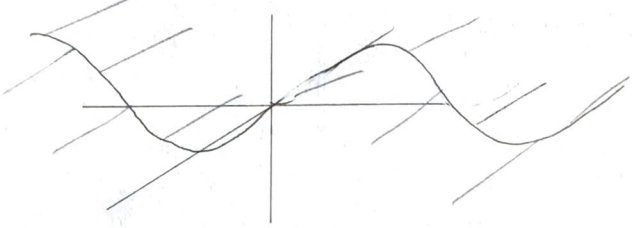
$C = x^2 + y^2$ the level curves are circles with radius \sqrt{c}



Ex. 4

Sketch

$$f(x,y) = \sin(y)$$

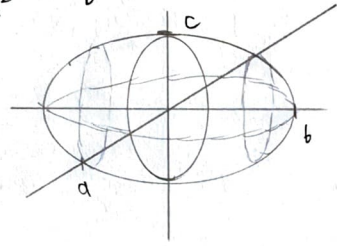


This is sometimes called a sheet

Ex. 5

Sketch

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

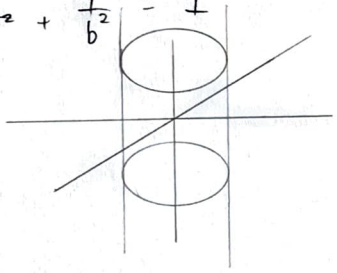


An ellipsoid

Ex. 6

Sketch

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



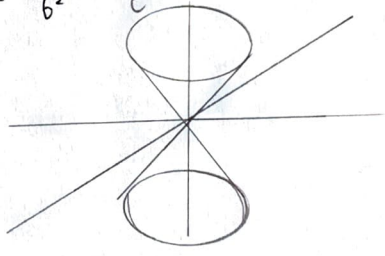
elliptic cylinder

if $a=b \rightarrow$ circular cylinder

Ex. 7

Sketch

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



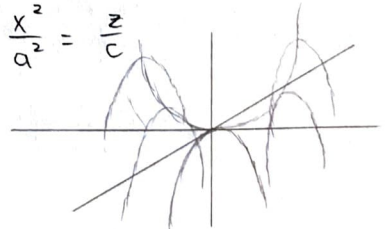
elliptic double cone

if $a=b \rightarrow$ circular double cone

Ex. 8

Sketch

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}$$



Hyperbolic paraboloid

Wow! This is awfull!